Supplementary Information

Computing the rate of convergence from experimental measurements

Given a time series of $\theta(t)$, the rate of convergence to the synchronization floor is defined as the exponent μ of the exponential decay to the floor, $(\theta - \theta_0) \sim \exp(-\mu t)$. This rate can in principle be determined from the eigenvalues of the coupling matrix scaled as $\{\frac{\epsilon}{d}\lambda_i\}$ and the master stability function, which is defined by the node dynamics, coupling function, and synchronous state.

In practice, to avoid problems with zero crossings, we perform a moving average over a small time interval on $\theta(t)$ to form $\langle\theta(t)\rangle$, the smoothed synchronization error. We then calculate μ by fitting $\langle\theta(t)\rangle$ to an exponential over a fixed time interval from 0.5 to 2.0 ms, which, for the networks considered in our experiments, is the typical time of transient to synchronization.

For each network, our statistics shown in Fig. 2(c) and Fig. S1(b) are based on performing this measurement for 100 independent realizations of the initial conditions.

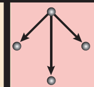
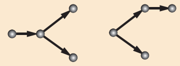
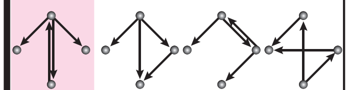
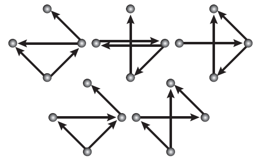
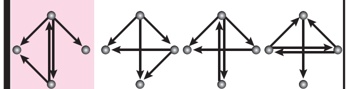
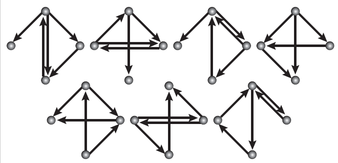
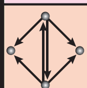
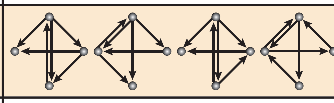
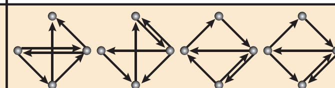
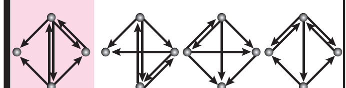
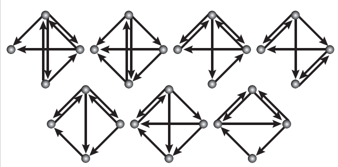
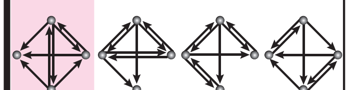
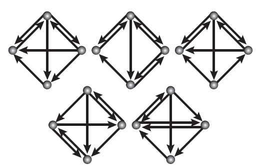
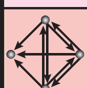
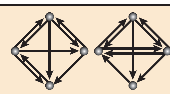
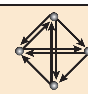
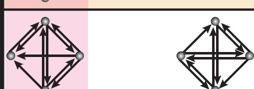
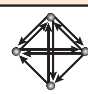
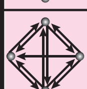
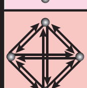
m	nonsensitive networks	sensitive networks		Λ	
	$g_d = 1$	$g_d = 2$	$g_d = 3$		
3				{0,1,1,1}	optimal
4				{0,1,1,2}	suboptimal
5				{0,1,2,2}	
6				{0,2,2,2}	optimal
7				{0,2,2,3}	suboptimal
8				{0,2,3,3}	
9				{0,3,3,3}	optimal
10				{0,3,3,4}	suboptimal
11				{0,3,4,4}	
12				{0,4,4,4}	optimal

TABLE S1. Structure of all optimal (shaded rows) and suboptimal (white rows) binary networks with $N = 4$ nodes. Each network is classified according to the number of connections m (rows) and geometric degeneracy g_d (columns). The highlighted column (leftmost) shows a path from an optimal tree ($m = 3$) to a fully connected network ($m = 12$) which contains only nonsensitive configurations ($g_d = 1$). The highlighted networks correspond to those used in the experiment of Fig. 2(c).

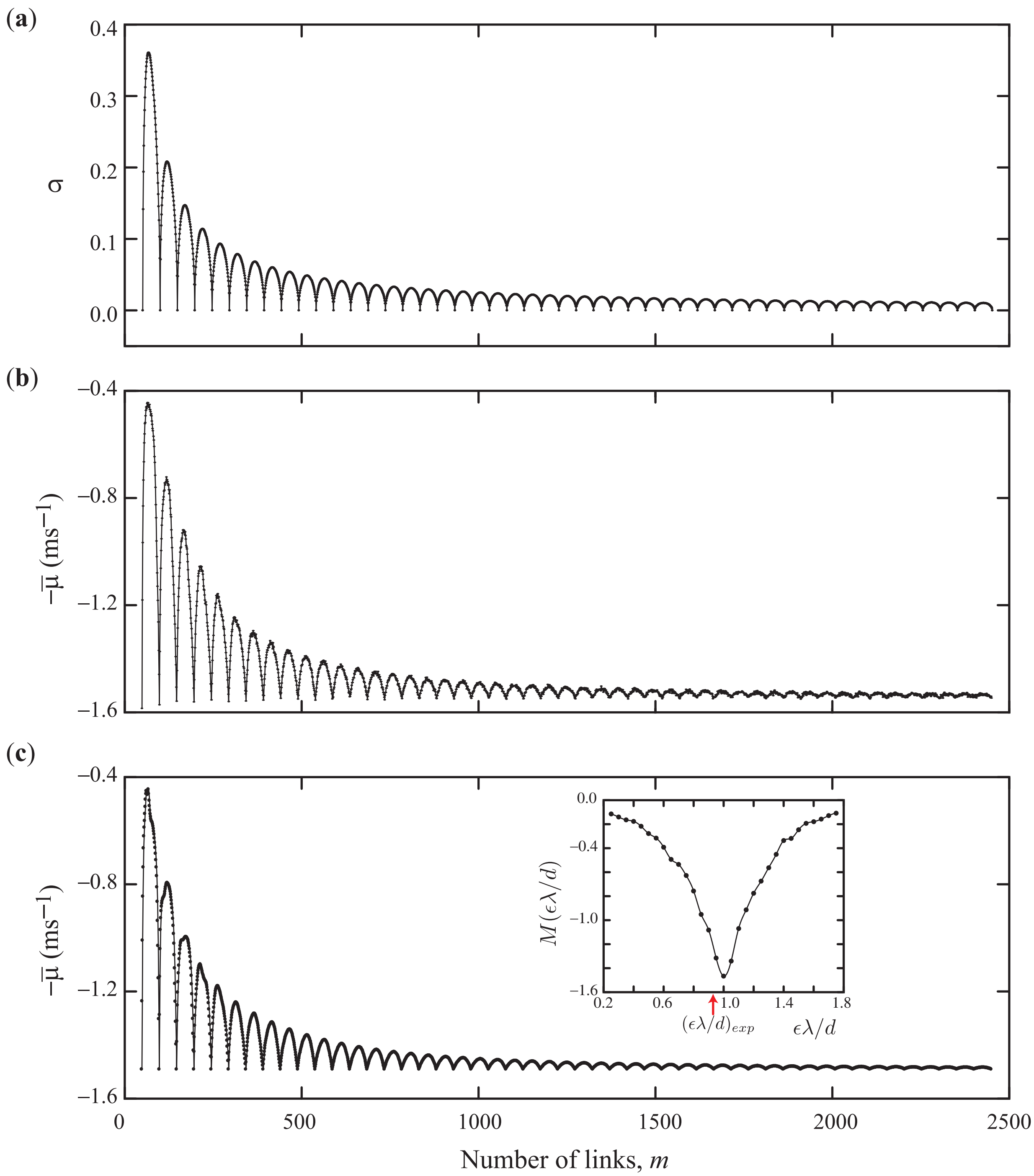


FIG. S1. Non-monotonic behavior of synchronization properties for nonsensitive networks with $N = 50$ nodes. (a) Eigenvalue spread σ as a function of the number of links m. (b) Mean convergence rate to synchronization, $\bar{\mu}$, determined by simulating the system as modeled by Eqs. (1)-(2) (main text). (c) Convergence rate estimated as a weighted average over the various eigenmodes using the experimentally measured master stability function $M(\epsilon\lambda/d)$ (inset), which was obtained from a series of experimental measurements on a two node network. The domain of M shown corresponds to the region where synchronization is observed experimentally. Each eigenvalue λ_i of the coupling matrix corresponds to a decay rate $\mu_i = M(\epsilon\lambda_i/d)$ in the direction of its associated eigenvector. Assuming that the coupling is enabled at $t = 0$ and the initial synchronization error is distributed evenly among all eigenmodes, the convergence rate μ satisfies $C\exp(-\mu t) = \sum_i \exp(-\mu_i t)$, where C is a normalization constant. Differentiating with respect to t leads to the estimate $\mu = \sum_i \mu_i \exp(-\mu_i t)/\sum_i \exp(-\mu_i t)$, which is shown in the figure for $t = 2.0$ms (this is a typical time for the experimental system to converge to the synchronization floor). The coupling strength ϵ used in (b, c) is chosen so that $\epsilon\lambda/d$ for the optimal networks of size 50 is the same as the experimental value used for the optimal networks of size 4. This is marked with the subscript *exp* in the inset of panel (c).

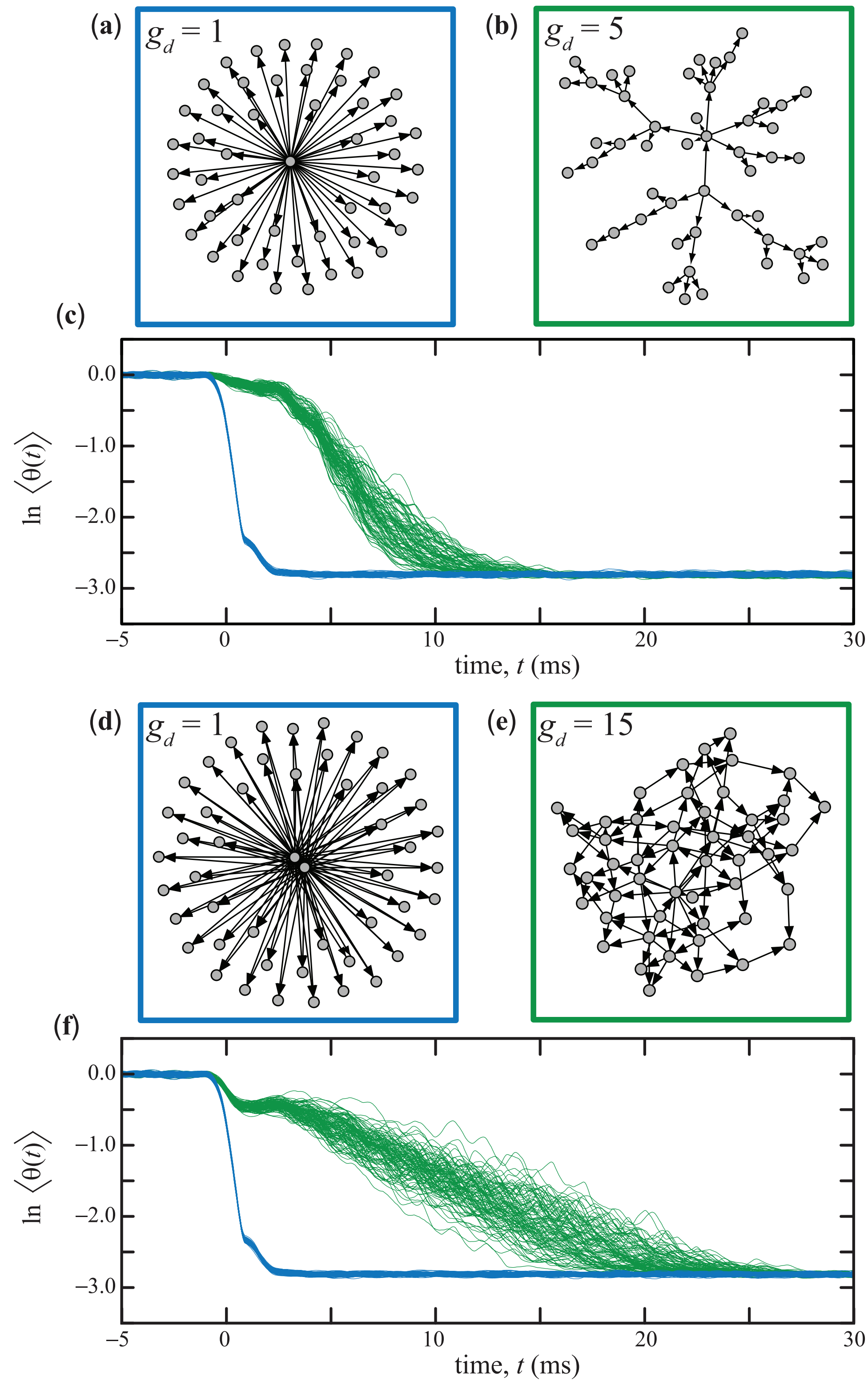


FIG. S2. Transition to synchronization for sensitive and nonsensitive networks with $N = 50$ nodes. (a, b) Configurations with $m = N - 1$ links and $g_d = 1$ and 5, respectively. (c) Numerical simulation of the synchronization error, $\langle\theta(t)\rangle$, for the networks shown in (a) ($g_d = 1$, blue) and (b) ($g_d = 5$, green). (d-f) Same as in (a-c) for configurations with $m = 2N - 2$ links, and $g_d = 1$ (blue) and 15 (green).

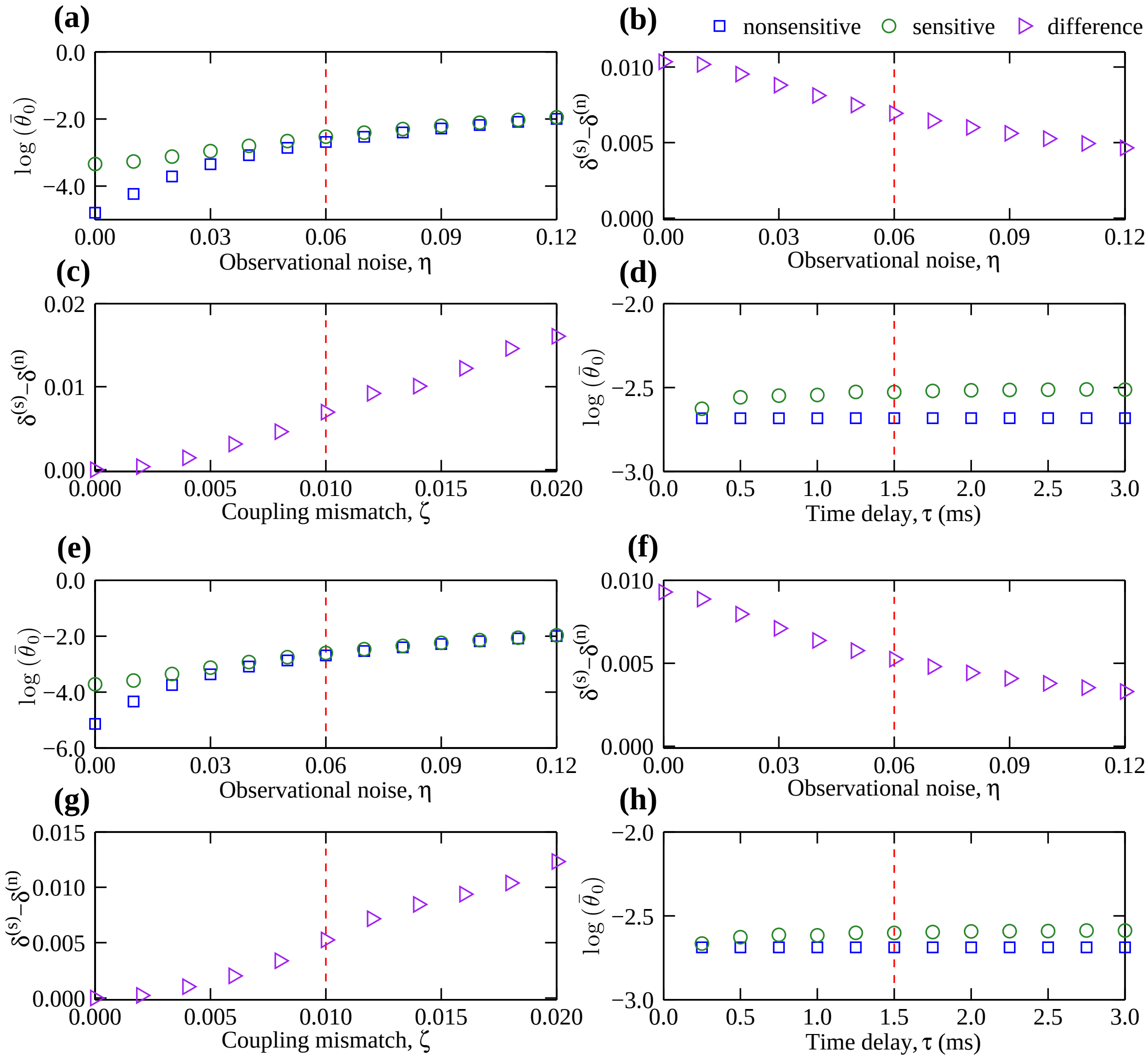


FIG. S3. Simulation results for networks with $N = 50$ nodes. (a-d) Counterpart of Fig. 4 for the networks considered in Fig. S2(a-c). (e-h) Corresponding results for the networks considered in Fig. S2(d-f).